Suma de las potencias p-ésimas de los primeros n números naturales.

Sean $p \in \mathbb{N} \cup \{0\}$ *y* $n \in \mathbb{N}$*:*

$$1^p + 2^p + \cdots + n^p = \frac{n^{p+1}}{p+1}\left(T_0 n^{-0} - T_1 n^{-1} + T_2 n^{-2} - \cdots + (-1)^p T_p n^{-p}\right)$$

Donde,

$$T_k = \begin{cases} 1, & si\ k = 0 \\ -\dfrac{a_0(k)T_0 + a_1(k)T_1 + \cdots + a_{k-1}(k)T_{k-1}}{a_k(k)}, & si\ k > 0 \end{cases}$$

Y, para $i \in \mathbb{N} \cup \{0\}$ *y* $k \in \mathbb{N}$, *con* $i \leq k$,

$$a_i(k) = \binom{p+1-i}{k+1-i}$$

También,

$$T_k = \begin{cases} 1, & si\ k = 0 \\ -\dfrac{a_0(1)T_0}{a_1(1)}, & si\ k = 1 \\ -\dfrac{a_0(2)T_0 + a_1(2)T_1}{a_2(2)}, & si\ k = 2 \\ -\dfrac{a_0(k)T_0 + a_1(k)T_1 + a_2(k)T_2 + a_4(k)T_4 + \cdots + a_{k-2}(k)T_{k-2}}{a_k(k)}, & si\ k(>2) es\ par \\ 0, & si\ k(>1) es\ impar \end{cases}$$